

Name: _____

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Math 12 Honours: Section 5.2 What are Logs and Basic with Logarithm

$$\log(A \times B) = \log A + \log B \quad \log\left(\frac{A}{B}\right) = \log A - \log B \quad \log A^n = n \log A \quad \log_a b^c = \frac{\log b^c}{\log a}$$

1. Rewrite each of the following in exponential form:

a) $\log_3 81 = 4$	b) $\log_4 2048 = b$	c) $-\log_5 a = 13$
d) $\log_c d = e$	e) $\log_{2x} 100 = 5$	f) $\log_{\sqrt{3}} a = b$

2. Evaluate each of the following without using a calculator:

a) $\log_2 8^3$	b) $\log_3 \sqrt{243}$	c) $\log 100 + \log_3 \sqrt{3}$
d) $3 \log_{0.125} 32$	e) $\log_2 (\log(0.0001))^2$	f) $\log_8 \left(\frac{\sqrt{256} \times 64}{\sqrt[3]{1024}} \right)$
g) $\log_5 25 \times \log_5 0.2 \times \log_{0.2} 125$	h) $\log_{12} 16 + \log_{12} 9$	i) $\log_{11} (2 \log_4 2^{51} + \log 10)$

3. Expand and rewrite each expression in terms of $\log A$, $\log B$, and or $\log C$

a) $\log\left(\frac{AB}{C}\right)$	b) $\log(A^3 \sqrt{B} \times C^{-1})$	c) $\log\left(\frac{10A}{\sqrt{B}}\right) - 0.5 \log(100C)$
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d) $\log(0.001\sqrt{ab})$	e) $\log\left(\frac{100a}{\sqrt{c^3b^4}}\right)$	f) $\log\left(\frac{\sqrt{10b}}{a^{3/2}}\right) - \log c^{-1}$
g) $\log\left(\frac{a^{-2}}{1000b^2}\right)$	h) $\left[\log_c\left(\frac{a^2}{b^3}\right)\right]^{-1}$	i) $\frac{\log_a b}{\log_c a} \times \frac{\log_b ac}{\log_b c}$

4. Express each of the following as a single logarithm

a) $2\log a + 5\log b$	b) $3\log x + \frac{1}{2}\log y$	c) $2\log a + \log b - 5\log c$
d) $\frac{1}{2}\log x - \frac{2}{3}\log y$	e) $3\log a - 4\log b - 0.5\log c$	f) $10\log a - \frac{3\log b}{2}$
f) $\frac{2\log a}{3\log b} + \frac{5\log b}{2\log b}$	g) $\frac{\log a}{0.4\log c} + \frac{\log b}{0.5\log c}$	h) $3\log\sqrt{a} + 2\log\sqrt[3]{b} - 3$

5. If $\log_3 2 = x$, simplify each logarithm in terms of "x"

a) $\log_3 8$

b) $\log_3 \sqrt{2}$

c) $\log_3 24$

d) $\log_3 18\sqrt{8}$

6. Given $\log_3 x = 2$ and $\log_3 y = 5$, evaluate each logarithm

a) $\log_3 xy$

b) $\log_3 (27x^3y)$

c) $\log_3 \left(\frac{3x}{y^3} \right)$

d) $\log_3 (810x^{-2}y)$

7. Use the fact that $\log_a b = \frac{\log_x b}{\log_x a}$ to simplify the following:

a) $(\log_x y)(\log_y x)$

b) $(\log_5 8)(\log_8 7)(\log_7 5)$

c) $\left(\frac{1}{\log_d x} \right) \left(\frac{1}{\log_c x} \right)$

d) $(\log_4 a)(\log_a 2a)(\log_{2a} x)$

$$e) \frac{\log_x a}{\log_{xy} a}$$

$$f) \frac{\log_c m}{\log_{cd} m} - \frac{\log_c m}{\log_d m}$$

$$8. \text{ Solve for "y" } (\log_3 x)(\log_x 2x)(\log_{2x} y) = \log_x x^2$$

$$9. \text{ If } \log x^5 y^3 = 25 \text{ and } \log \frac{x}{y} = 3, \text{ then what is the value of } \log x ?$$

$$10. \text{ Prove the following identity: } \frac{1}{\log_a b} + \frac{1}{\log_c b} = \frac{1}{\log_{ac} b}$$

$$11. \text{ Find the value of } \sum_{n=1}^{999} \log \sqrt[3]{\frac{n^2}{n^2 + 2n + 1}}$$

$$12. \text{ Simplify the product completely: } \frac{\log_2 3}{\log_4 3} \times \frac{\log_4 5}{\log_6 5} \times \frac{\log_6 7}{\log_8 7} \times \dots \times \frac{\log_{124} 125}{\log_{126} 125} \times \frac{\log_{126} 127}{\log_{128} 127}$$

13. If $\log(xy^3) = 1$ and $\log(x^2y) = 1$. What is $\log(xy)$? Amc 2003 12A

14. If $a \geq b > 1$, what is the largest possible value of $\log_a\left(\frac{a}{b}\right) + \log_b\left(\frac{b}{a}\right)$? AMC 2003 12B

15. How many distinct four-tuples (a, b, c, d) of rational numbers are there with:

$$a \log_{10} 2 + b \log_{10} 3 + c \log_{10} 5 + d \log_{10} 7 = 2005$$

Amc 2005 12B

16. The numbers $\log(a^3b^7)$, $\log(a^5b^{12})$, $\log(a^8b^{15})$ are the first three terms of an arithmetic sequence, and the 12th term of the sequence is $\log(b^n)$. what is the value of "n"? amc 12

17. Two distinct numbers "a" and "b" are chosen randomly from the set $\{2, 2^2, 2^3, \dots, 2^{26}\}$. What is the probability that $\log_a b$ is an integer? AMC 12 modified